Faster and More Efficient Mixed Logit Models through Generalized Antithetic Draws and Double Base Shuffling

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There is growing interest in estimating larger and more flexible discrete choice models in the following contexts: a. simultaneous modeling of several interdependent activity and travel dimensions, b. dynamic models of repeated choices of individuals over time (involves \( n_{alt} \times n_t \) dimensions), c. models of unobserved heterogeneity (with many random taste parameters). Mixed Logit models based on Quasi-Random draws (especially Halton draws) are widely used to estimate such models.

Existing mixed logit models have mainly been used for discrete choice contexts with up to 10-20 alternatives. For larger dimensional contexts, however, currently used mixed logit models are still too expensive for practical applications. Furthermore, the performance of Halton draws degrades with increasing dimensionality. To overcome these limitations, shuffling and scrambling methods to improve Halton draws and non-Halton draws have been proposed, but at a significant increase in computational cost.

This paper investigates three related objectives: First, we propose an improved method for generation of Halton draws (QMCGA) for mixed-logit models that combines the use of better sampling (generalized antithetic draws) and better shuffling techniques. The second objective is to investigate the relative performance of QMCGA for likelihood estimation and probability evaluation relative to alternative generation techniques under varying dimensions and probability levels. The final objective is to analyze the performance of the QMCGA draws in terms of likelihood maximization in relation to standard Halton and pseudo random draws.

The proposed technique relies on the application of two key ideas: a) generalized antithetic draws and b) double-base shuffling method. The generalized antithetic draw is obtained as a logical extension to the simple antithetic draw at two levels. The antithetic variates are used at both the uniform halton draw generation level and at the standard normal variate generation level. Further, unlike the simple antithetic draws, where only two normal vectors (\( Z \) and \( -Z \)) are obtained from a single normal vector generation (\( Z \)), upto \( 2^m \) standard normal vectors may be obtained from a single vector \( Z \) by permuting the components. For this study, a smaller subset with \( m \) vectors is generated from a single \( Z \) vector, resulting in an \( m \)-fold reduction in the generation time. The use of antithetic draws further increases the precision of the probability estimates.

To address the problem of dimensional degradation, the double base shuffling (DBSH) approach is proposed. The principle behind the double base shuffling is to take advantage of the progressively uniform coverage that halton sequences provide, while at the same time not compromising on the quality as dimensionality increases. The standard halton sequences deteriorate with increasing dimensionality because of the long cycle lengths, incomplete cycles (leading to biases) and correlation across different dimensions. These first two drawbacks are absent in the DBSH because the cycle length of the base's is 2 and 3 (as only base 2 and base 3 sequences are used). The correlations across dimensions
(which is already low since the cycle length is less) is further reduced by shuffling the obtained sequences.

The effectiveness of the proposed technique in terms of probability evaluation is investigated by conducting computational experiments. Twenty different synthetic input sets (sys utility and var-cov matrix, randomly generated) were used and 10 replications were performed for each input set. The computation experiments are conducted for a range of probability values (low: 0-0.2, medium 0.4-0.6, and high 0.8-1.0) and for Small (5d) and large dimensions (15d), and a wide range of draws (200 - 6400). The proposed method is compared against QMC regular and pseudo-monte-carlo draws, and the performance quantified using RMS error, precision and computation times.

The results show that for the five-alternative choice context, the QMCGA is 3-4 times faster than corresponding QMC regular (for a fixed level of accuracy – i.e. RMS error in probability ranging from 2 to 0.06%). For the higher dimension (15d), the proposed QMCGA with DB is 1.9-5.8 times faster than pseudo-random draws (PMC) which outperforms QMC regular due to the dimensional degradation. Although QMC with standard shuffling was faster than QMC, it was significantly slower than QMCGA. For a given computational time (0.1-10s per evaluation), QMCGA is more accurate than PMC and QMC for both 5 and 15 dimensional choice contexts. The RMS errors of the QMCGA were 2.3-2.9 times smaller than standard halton draws for 5d, and 2–6 times smaller for the 15 d case.

The rate at which the accuracy increases (rms error R) with increasing draws (N) is also studied by calibrating the equation \( R = aN^b \). The power coefficient \( b \) represents the asymptotic convergence rate, while the constant ‘a’ indicates the convergence speed at lower number of draws. For the 15dimension scenario, the QMC regular deteriorates drastically (the initial convergence coefficient ‘a’ is 10-30 times worse than QMCGA and 8-10 times worse than PMC). However, the asymptotic convergence rate is higher (\( b = -0.8 \) to -0.85), whereas, the asymptotic convergence rate of QMCGA reduces to some extent (\( b = -0.6 \) to -0.7) for the higher dimension.

In the third objective, the likelihood optimization performance of QMCGA is analyzed by comparing the computational time, estimate accuracy and precision, and likelihood at convergence against standard Halton draws. The likelihood optimization is performed for a range of dimensions and error structures: 5d and 15d (only preference heterogeneity) and 15d and 30d (with both preference and response heterogeneity). It was found that both QMC regular and QMCGA produce comparable converged likelihood values and estimates. The computational time for QMCGA were significantly lower for all cases. QMCGA offers a speed-up of 3.7 to 2.5 fold (for 5 and 15 d preference heterogeneity case respectively). The speed improvement factors with both preference and response heterogeneity were 4.7 and 3.5 (for 5d and 15d respectively). In terms, of number of halton draws needed, the QMCGA needs 3-5 times fewer draws than the regular QMC to achieve a comparable converged LL value and coefficient estimates. The results show that the proposed QMCGA offers significant promise of faster mixed logit models and reduced dimensional degradation for large dimensional choice contexts.